P: ISSN No. 2231-0045 RNI No. UPBIL/2012/55438 VOL.-III, ISSUE-I, August-2014
E: ISSN No. 2349-9435
Periodic Research

2D Geometric Transformations in Computer Graphics

Abstract

2D Geometric Transformations are used to change the shape, size position and direction of objects like lines, different types of curves such as circles, different forms of ellipse etc in computer Graphics. All such graphic objects are constructed with finite number of points. A point itself has no geometric and analytic properties other than its position .Though it doesn't have any other geometric characteristics but for object transformation in computer graphics, it matters most. In geometric transformations every object is assumed as a collection of points. Every object point P has coordinates (x, y) and so the object is totality so all such points. Because of all sorts of object transformations. Scaling, Translation, Rotation etc results from simultaneous transformations. Here in this paper, an attempt has been made to demonstrate some applications of 2D geometric transformations in computer Graphics.

Keywords: 2D Geometric Transformation, Rotation Equations, Circle Equations.

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Introduction

In this article it is proposed to study the 2d geometric transformations with the help of rotation equations for any pivot point. The equations for a pivot point rotation are:

$$\left.\begin{aligned} X' &= x + (x1-x)\cos\theta - (y1-y)\sin\theta \\ Y' &= y + (x1-x)\sin\theta + (y1-y)\cos\theta \end{aligned}\right\} \text{------------- (1)}$$

And we also compare these rotation equations with the equations

$$\left.\begin{aligned} X' &= x + r\cos\theta \\ Y' &= y + r\sin\theta \end{aligned}\right\} \text{--------------------------------- (2)}$$

When a line is rotated through angles from 0 degree to 360 degree using equations (1) the point moves and a circle is obtained on the computer screen. Similarly when a line is rotated using equations (2) points are located at different positions on the screen but the same circle is obtained on the computer screen. However for vertical and horizontal lines we obtained the same result.

Also when

$$\left.\begin{aligned} X' &= x + (rx)\cos\theta - (rx)\sin\theta \\ Y' &= y + (ry)\sin\theta + (ry)\cos\theta \end{aligned}\right] \text{-------------- (3)}$$

The rotation is performed using equation (3) we find different forms of ellipse.

Results

The Rotation Equations are:

$$\left.\begin{aligned} X' &= x\cos\theta - y\sin\theta \\ Y' &= x\sin\theta + y\cos\theta \end{aligned}\right\} \text{.............................. (4)}$$

Obviously (X' , Y') are the coordinates of the points when rotated through an angle θ relative to a set rectangular coordinate axes. How ever if the

the object is placed at an arbitrary point whose coordinates are (X', Y') relative to the old set of axes and is rotated there after than the equations are given by:

X'= x + (x_1-x) cosθ – (y_1-y) sin θ

Y'= y + (x_1-x) sinθ + (y_1-y) cos θ

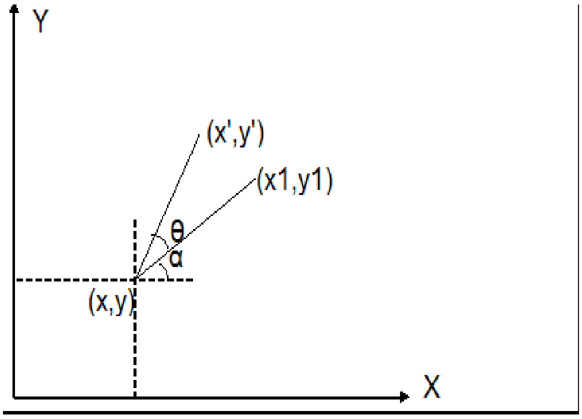

Fig – 1.0

The equations given below can also be used for the rotation of a line.

X'= x + r cos θ

Y'= y + r sin θ

We find that these equations rotate the object on a circular path. It is obvious that the length of the line remains the same throughout the rotation. The (radius) r is used as the length of the line remains the same throughout the rotation. The (radius) r is used as the length of the line.

When we take x=100 and y= 100 and use the equations (iii) & (iv) and find the values for different angles also using x1=150 and y1= 150

X'= x + (x1-x)cos θ –(y1-y)sin θ

Y'= y + (x1-x)sin θ + (y1-y) cos θ

Fig – 1.1

Angle θ = 30 degree
X'= 118.309
Y'= 168.299
Angle θ = 60 degree
X'= 81.712
Y'= 168.304
Angle θ = 90 degree
X'= 50.014
Y'= 150.014
Angle θ = 120 degree
X'= 31.705
Y'= 118.328
Angle θ = 150 degree
X'= 31.689
Y'= 81.732
Angle θ = 180 degree
X'= 49.970
Y'= 50.029
Angle θ = 210 degree
X'= 81.651
Y'= 31.711
Angle θ = 240 degree
X'= 118.247
Y'= 31.684
Angle θ = 270 degree
X'= 149.955
Y'= 49.955
Angle θ = 300 degree
X'=168.283
Y'= 81.631
Angle θ = 330 degree
X'= 168.321
Y'= 118.327
Angle θ = 360 degree
X'= 150.049
Y'= 149.940

Now using the equations

X'= x + r cos θ

Y'= y + r sin θ

fig-1.2

Angle θ = 30 Degree
X'= 160.625
Y'= 134.994
Angle θ = 60Degree
X'= 135.011
Y'= 160.614
Angle θ = 90 Degree
X'= 100.020

Y'= 170.000
Angle θ = 120 Degree
X'= 65.020
Y'= 160.635
Angle θ = 150 degree
X'= 39.395
Y'= 135.029
Angle θ=180 degree
X'= 30.000
Y'= 100.041
Angle θ = 210degree
X'= 39.354
Y'= 65.041
Angle θ= 240 degree
X'= 64.952
Y'= 39.405
Angle θ = 270 degree
X'= 99.937
Y'= 30.000
Angle θ = 300 degree
X'= 134.940
Y'= 39.343
Angle θ = 330 degree
X'= 160.583
Y'= 64.934
Angle θ = 360 degree
X'= 169.999
Y'= 99.917

If we take X' =x + rx cos θ and Y'= y+ ry sin θ where rx ≠ ry an ellipse will be described with major axis horizontal and rx < ry and a vertical ellipse is described when rx > ry Verticle ellipse with rx < ry

Fig 1.3
Horizontal ellipse with rx > ry

Fig 1.4

Conclusion

In this paper we have chosen to study and compare the equations of rotation

X'= x + (x1-x)cos θ –(y1-y)sin θ
Y'=y + (x1-x)sin θ + (y1-y) cos θ
With the X'= x + r cos θ
and Y'= y + r sin θ

We have noticed that the rotation equations for the pivot point can rotate the line on computer screen and the above equations can also rotate the same but the points obtained on the same circular path are different

However if we take the horizontal line and vertical line then same result obtained by both the equations

Further if we take rx ≠ ry then an ellipse is obtained on the computer screen.

It is interesting to note that if in the equations

X'= x + (x1-x)cos θ –(y1-y)sin θ
Y'=y + (x1-x)sin θ + (y1-y) cos θ
(x1-x) = (y1-y) then once again a circle is obtained.
If we take the above equations as :-
X'= x + (rx) cos θ – (rx) sin θ
Y'=y + (ry) sin θ + (ry) cos θ

Then a horizontal ellipse is noticed on the computer screen if rx > ry similarly a vertical ellipse is noticed if rx< ry.

The purpose of making this research paper is to study and justify the importance of 2D geometric Transformations in computer Graphics.

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